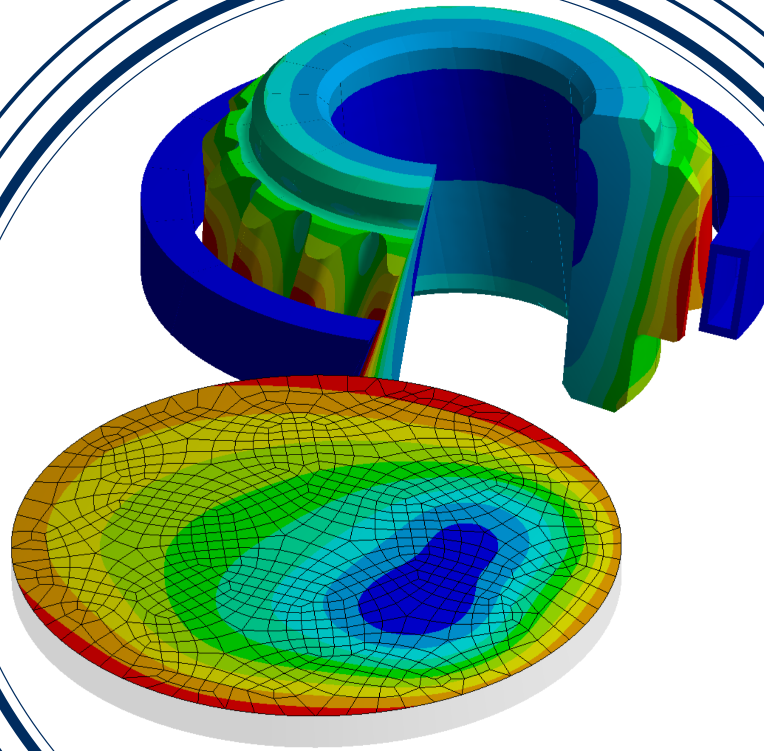
Back to transient - how to reduce coupled field transient nonlinear models for system level simulations

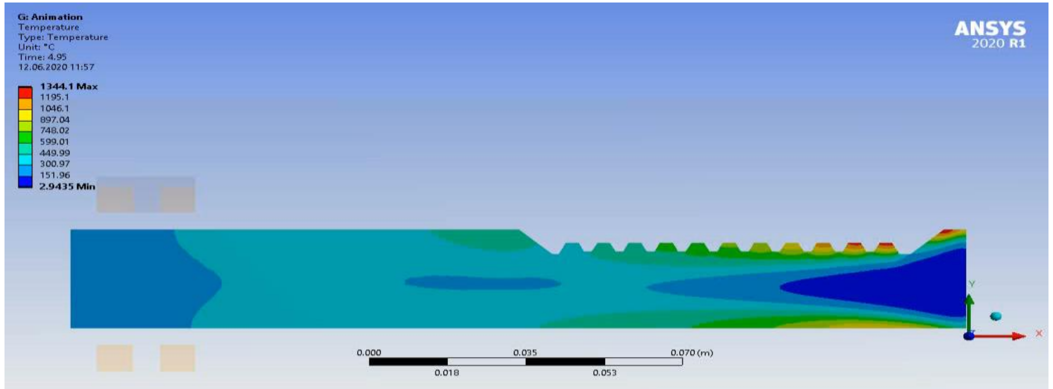
Hanna Baumgartl
Martin Hanke



CADFEM® Ansys CERTIFIED ELITE CHANNEL PARTNER

Motivation: Process parameter control for inductive hardening

- Process involves interaction of several physical domains:
  - Electromagnetic
  - Thermal
  - Structural (including phase transition, …)



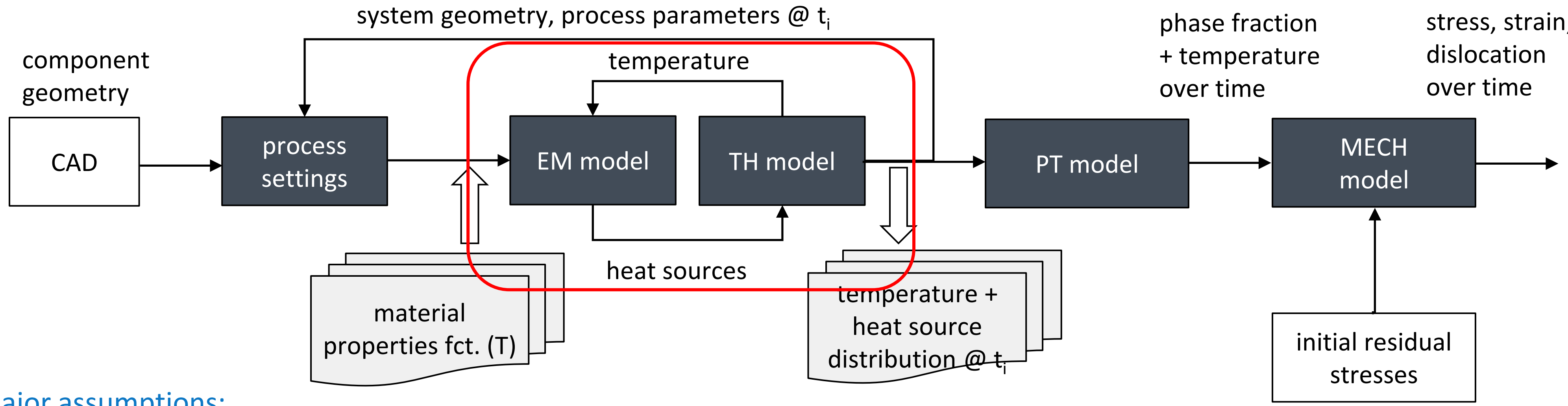
- Large number of process parameters
- Nonlinear, time depending interaction
- Interaction across several process steps

- Known issues:
  - Distortion of the components
  - Distortion spread highly sensitive to process parameters, material combinations, ….

- Existing Workflow on field level:
  - Good results, but too slow for systematic variation of parameters
  - Far too slow for online monitoring of process parameters

Induction Hardening of Metals
Current Status: Model Structure

Model Structure:



Major assumptions:

Thermal and electromagnetic model sequentially coupled

Empirical material model

Implementation:

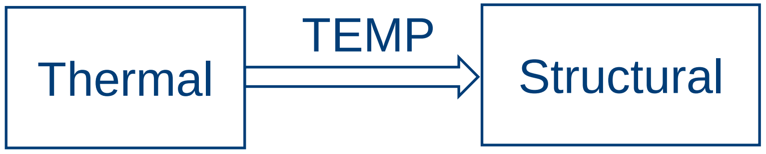
ANSYS workbench

- quantitative description of progressive (moving inductor) inductive hardening process possible
- description allows to quantify distortion, improve process development, provide basis for reliability assessment

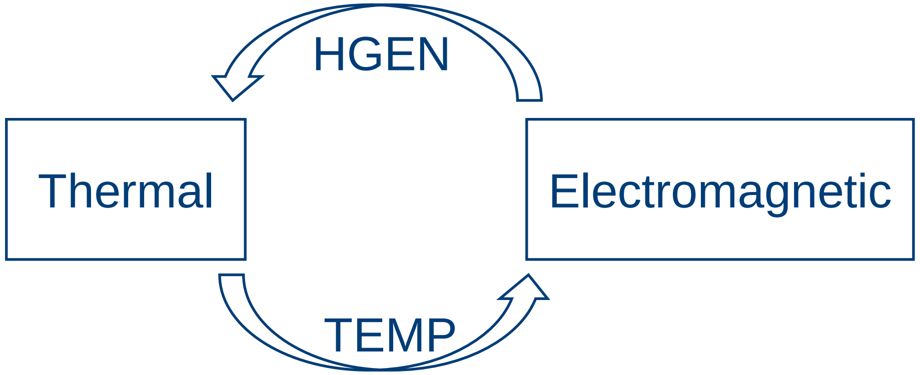
Source: Hamza Jamil „ Model Order Reduction – Induction Hardening Process", MORSS 8.9.2020

Interaction of physical Domains

Unidirectional coupling



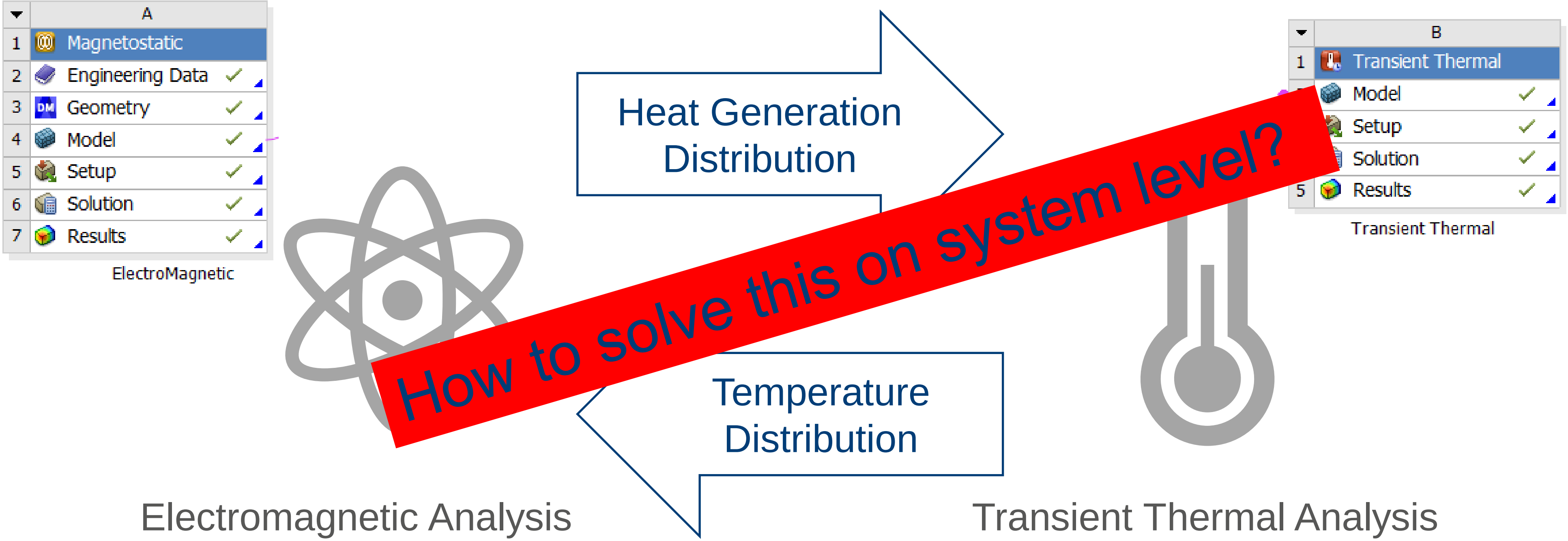
Bidirectional coupling



`LDREAD`

`UPGEOM`

Field Coupling



Electromagnetic Analysis

- **Static** interaction: actual temperature distribution gives actual heat generation
- **Nonlinear**: BH-curve, temperature dependent, position dependent

Transient Thermal Analysis

- **Transient** behaviour: last time step is start for next
- **Linear**: PDE sytem with constant coefficients

System simulation ≠ field simulation

CADFEM

Field level:

- Physics represented through:
  - Spatial discretization
  - Large number of distributed results (nodes/elements)
  - 3D

- Coupling:
  - On element / node level (Multiphysics elements)
  - On mesh level (Exchange of elemental/nodal data from domain to domain)

→ Exchange of field data

System level:

- Physics represented through:
  - Models: Meta / ROM / Analytical
  - Small number of concentrated results
  - 0D

- Coupling:
  - Through terminals (causal or conservative)
  - Averaged or integral data (e.g. remote points integral current / flow)

→ Exchange of (a few) scalar data

Characterization of spatially distributed quantities

CADFEM

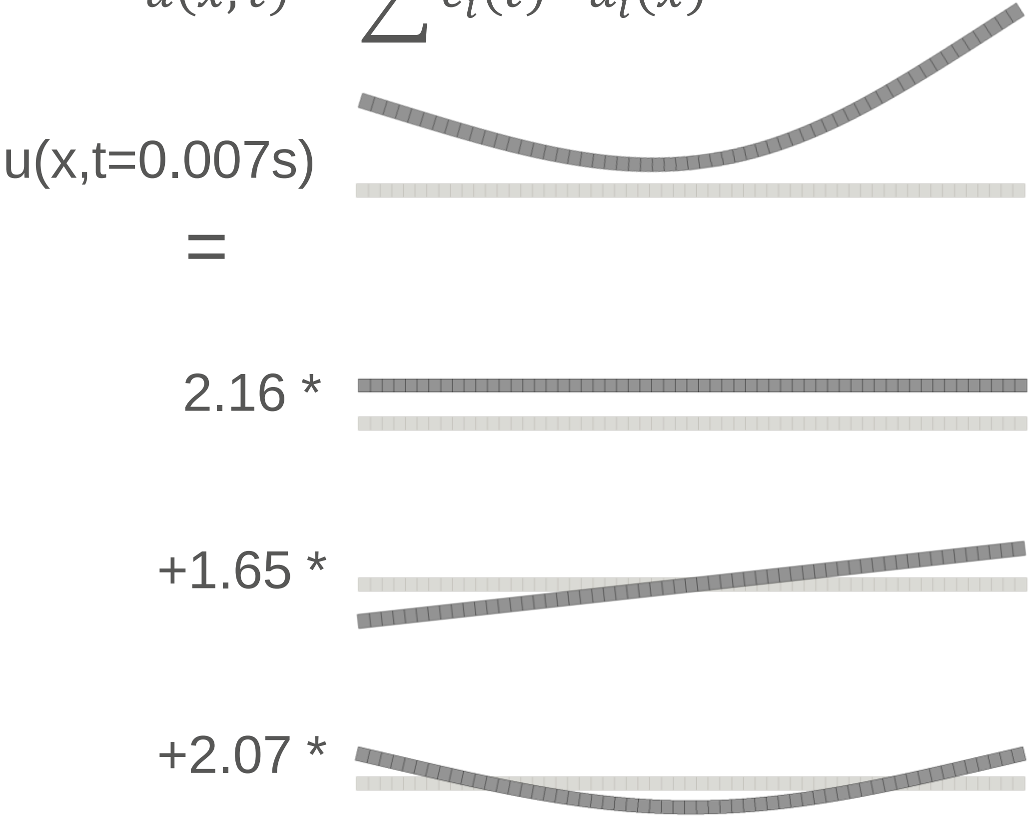
$$u(x,t) = \sum c_i(t) \cdot u_i(x)$$

- Temperatures and heat generation rates

- Approximation through polynomials:
  - Average
  - Averaged slope
  - Averaged curvature
  - …

- Example: Deformation
- Linear combination of basis deformations

- Generalization: Any orthogonal (orthonormal) basis



Projection: Determine coefficients

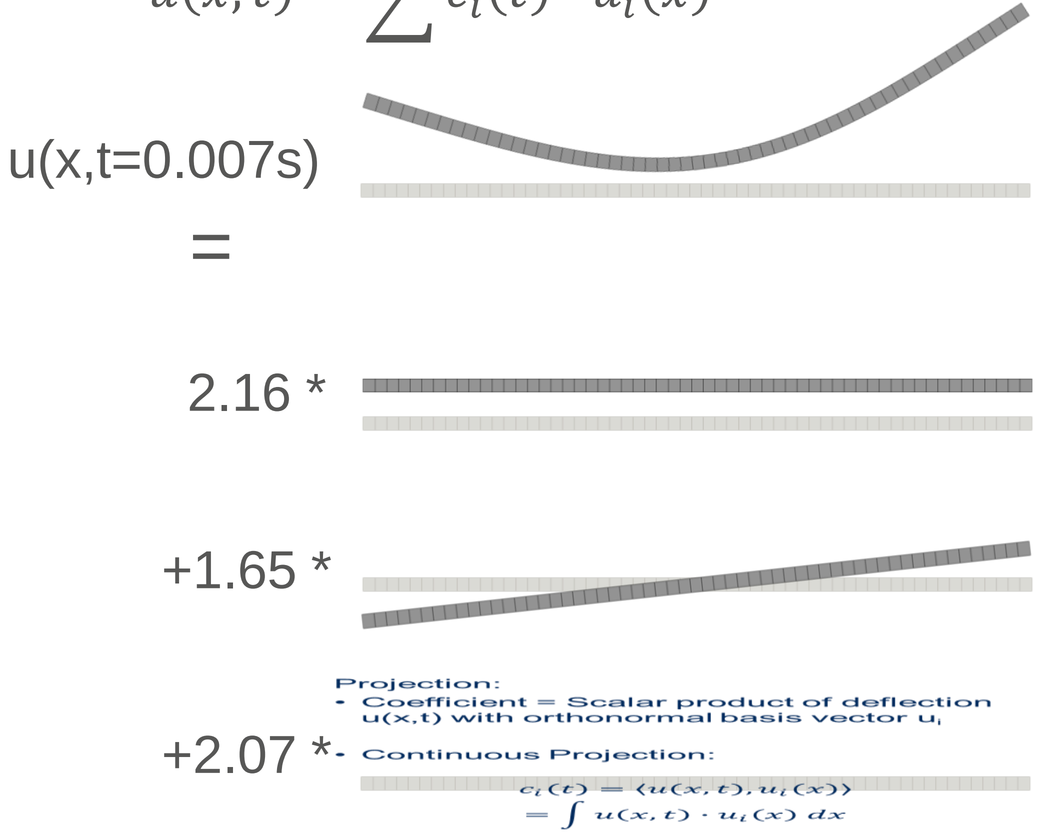
$$u(x,t) = \sum c_i(t) \cdot u_i(x)$$

Projection:

- Coefficient = Scalar product of deflection u(x,t) with orthonormal basis vector u_i

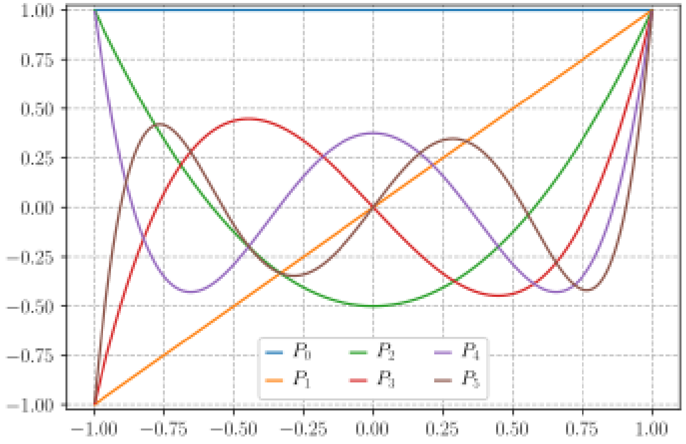
- Continuous Projection:

$$c_i(t) = \langle u(x,t), u_i(x) \rangle$$
$$= \int u(x,t) \cdot u_i(x) \, dx$$



Orthonormal systems

- 1D: Orthogonal polynomials:
  - Legendre/Chebyshev (bounded)
  - Fourier (periodically)



Legendre:

- Defined on an interval [-1,1]
- Defined to construct an orthogonal system:
- $\langle P_n, P_m \rangle = \int_{-1}^{1} P_n(x) \cdot P_m(x) dx = \delta_{n,m}$

- Norm (length) of each basis vector:
- $\|P_n(x)\|_2 = \sqrt{\int_{-1}^{1} P_n(x)^2 dx} = \sqrt{\frac{2}{2n+1}}$

→Orthonormal basis defined by

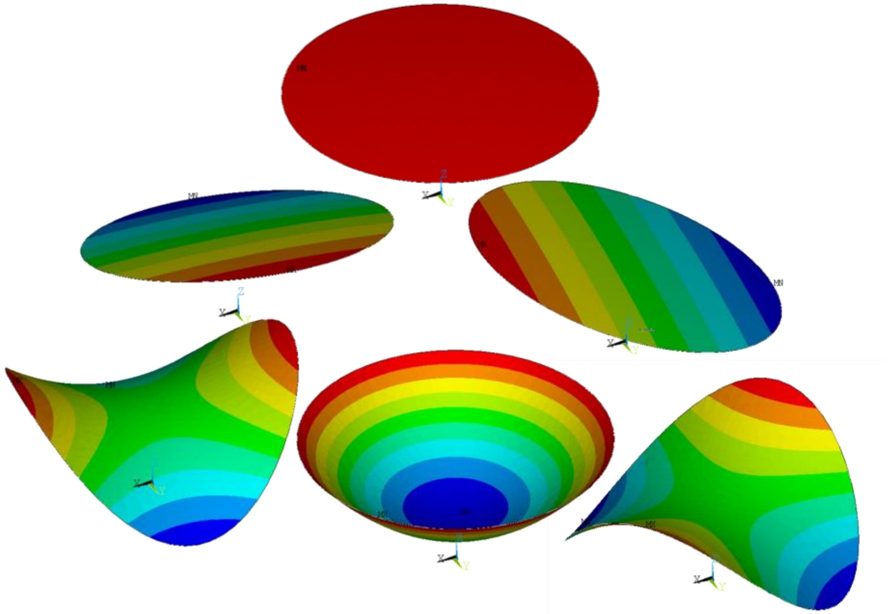
$$\frac{P_n(x)}{\|P_n(x)\|} = \frac{P_n(x)}{\sqrt{\frac{2}{2n+1}}}$$

Orthonormal systems

CADFEM

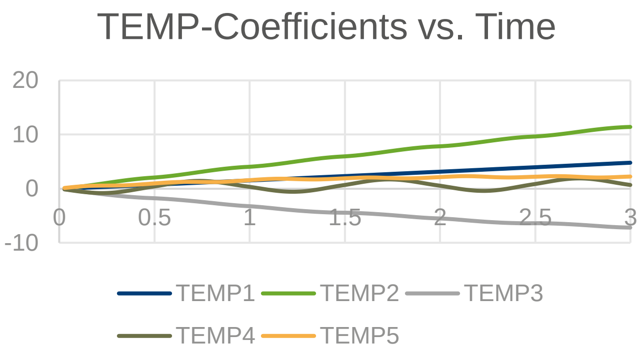
- 1D: Orthogonal polynomials:
  - Legendre/Chebyshev (bounded)
  - Fourier (perodically)

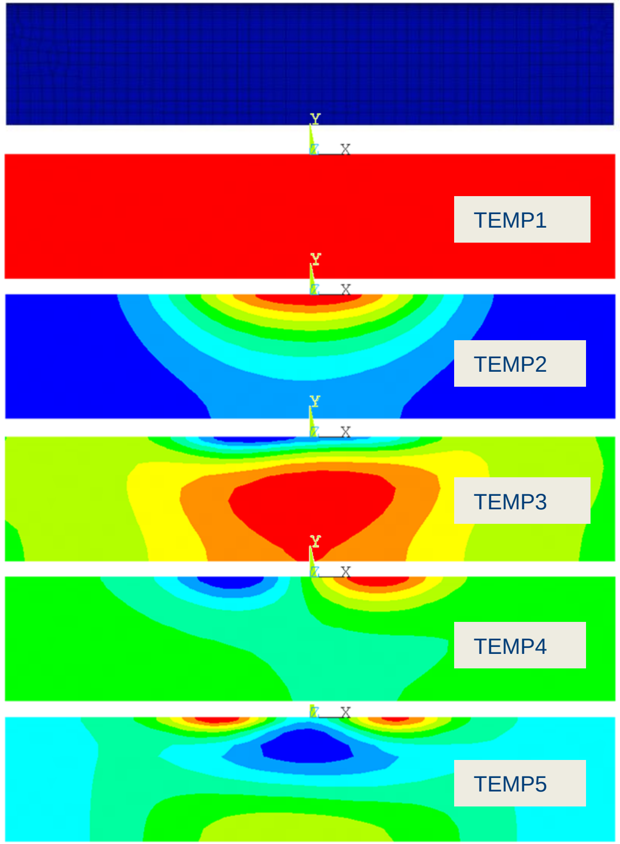
- 2D: Orthogonal polynomials
  - Zernike (defined on circle)
  - Spherical harmonics (defined on sphere surface)



Orthonormal systems

- 1D: Orthogonal polynomials:
  - Legendre/Chebyshev (bounded)
  - Fourier (perodically)
- 2D: Orthogonal polynomials
  - Zernike (defined on circle)
  - Spherical harmonics (defined on sphere surface)
- 3D: Modes:
  - Structural eigenmodes
  - Derived from orthogonalization (Krylov, SVD, MOS, POD,…)

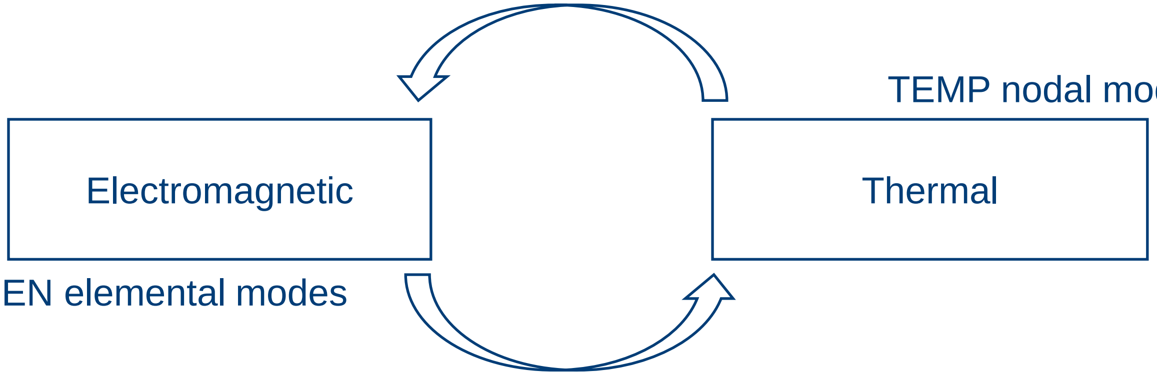




Which part of the solution is in place?

Coefficients of temperature distribution:
Orthogonal projection on nodes

TEMP nodal modes



HGEN elemental modes

Coefficients of distributed heat generation rates:
Orthogonal projection on elements

Domain behaviour and reduction

System response nonlinear stationary:

- Electromagnetic (periodically transient)
- Teaching: Fitting of computed samples

- Result: Look-up-table, response surface

System response linear dynamic:

- Structural, thermal
- Reduction: Modal, Krylov

- Result: State space models (SSM)

Which part of the solution is in place?

Coefficients of temperature distribution:
Orthogonal projection on nodes

TEMP nodal modes

EM: nonlinear stat.,
look-up-table

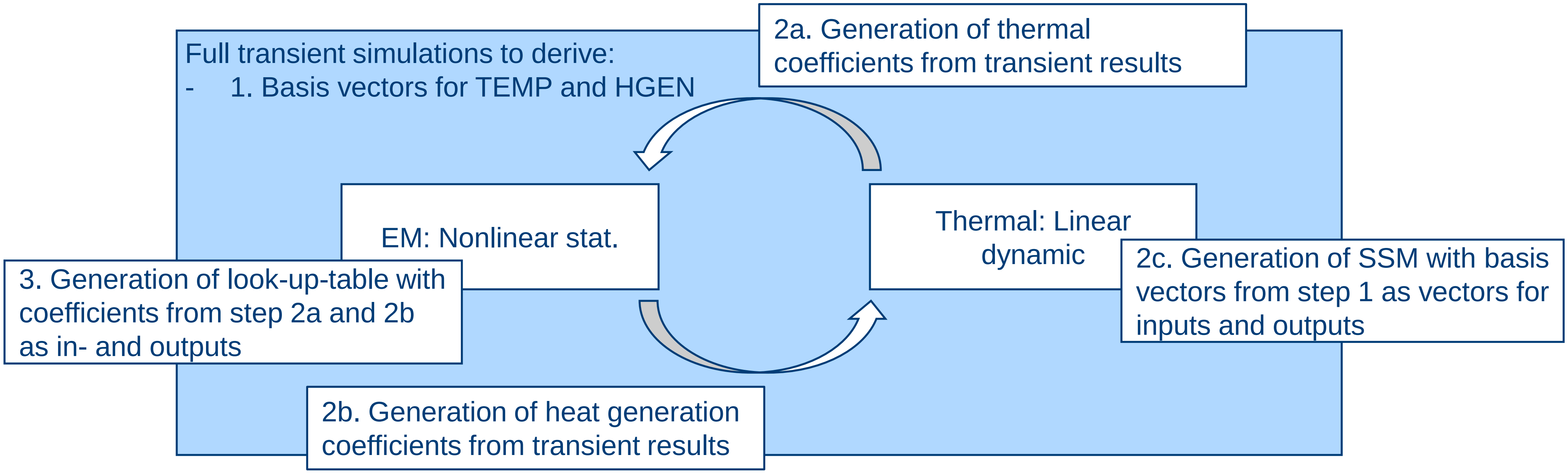
Thermal: Linear
dynamic, SSM

HGEN elemental modes

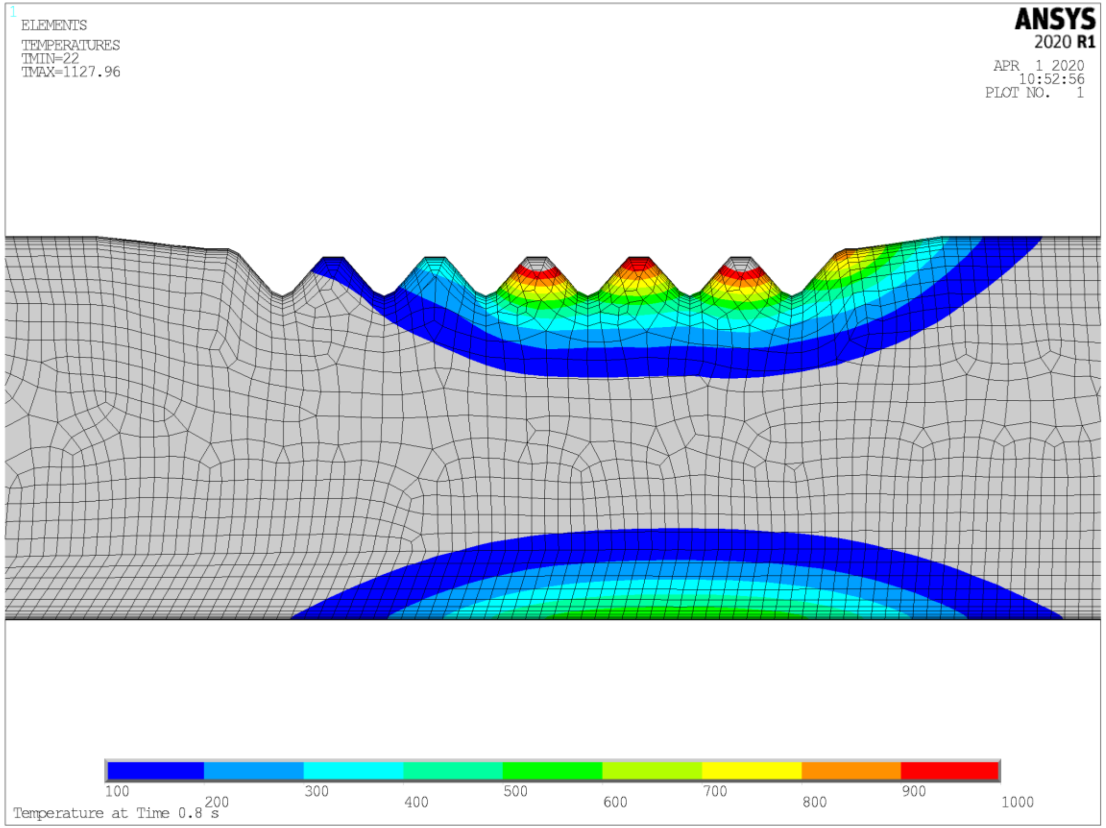
Coefficients of distributed heat generation rates:
Orthogonal projection on elements

Resulting workflow

CADFEM



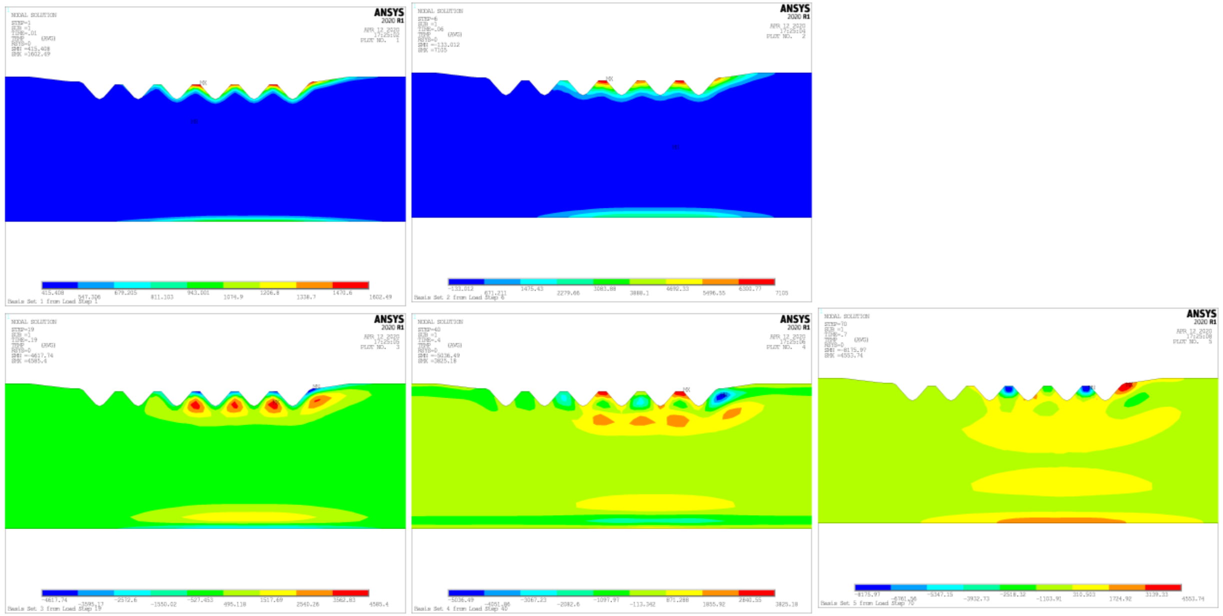
Generation of basis vectors
Transient temperature distribution



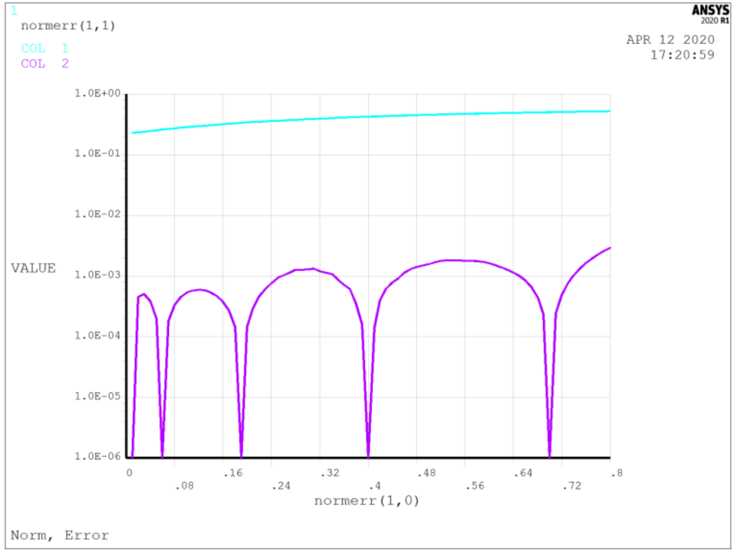
Temperature Distribution During Heating

Orthogonalization of snapshots taken over time

Basis vectors



Time evolution of coefficients:



Systematic approach: Method of snapshots (MOS)

- Wide range of technologically achievable parameters
- Large number of transient simulation results
- Systematic and automatized approach for basis vector generation required

- Method of snapshots:
- Modes constructed based on lagest eigenvalues of covariance matrix

Thesis MOR Inductive Hardening

MOS vs SVD

```python
import numpy as np
from numpy import linalg as LA
import scipy.linalg
import time
#################### Load Data #######################
mat = np.loadtxt('ndtemp_m.dat')
#################### SVD #############################
start_SVD = time.clock()
U, s, Vh = scipy.linalg.svd(mat,False) # For Compari
elapsed = time.clock()
time_elapsed_SVD = 1000*(elapsed - start_SVD)
#################### POD #############################
start_POD = time.clock()
mat_t=mat.T #transpose matrix
c=np.dot(mat_t, mat) #covariance matrix
#%solve eigenvalue problem - only depends on the num
w, v = LA.eig(c)
zeta = np.dot(mat,v)  #modes
for i in range(len(w)):
    zeta[:,i]=zeta[:,i]/LA.norm(zeta[:,i],2) #normal
elapsed = time.clock()
time_elapsed_POD = 1000*(elapsed - start_POD)
############## Comparison with SVD ##################
error_modes = LA.norm(zeta-U,2)
error_sigma = s - np.sqrt(w) #compare singular value
#####################################################
np.savetxt('U_h.txt', zeta, fmt='%20.12e')
np.savetxt('s.txt', np.sqrt(w), fmt='%20.12e')
```

Name	Typ	Größe	Wert
U	float64	(16564, 20)	[[-1.01287986e-03 -1.39089874e-03 1.49612865e-03 ... 2.45471855e-02 ...
Vh	float64	(20, 20)	[[-1.88145700e-01 -1.88145700e-01 -5.89928773e-02 ... -2.71261021e-01 ...
c	float64	(20, 20)	[[20603280. 20603280. 6465724. ... 29695492. 33603588. 37686187.] [2 ...
error_modes	float64	1	2.073741085710569
error_sigma	float64	(20,)	[-7.27595761e-12 -1.87583282e-12 7.74491582e-12 ... -1.03870318e+00 . ...
mat	float64	(16564, 20)	[[5. 5. 2. ... 7. 7. 8.] [1. 1. 1. ... 2. 2. 2.]
mat_t	float64	(20, 16564)	[[5. 1. 3. ... 0. 0. 0.] [5. 1. 3. ... 0. 0. 0.]
s	float64	(20,)	[2.41216749e+04 3.84506805e+02 8.64668825e+01 ... 1.90984829e+01 1.86 ...
time_elapsed_POD	float	1	3.6384999999938827
time_elapsed_SVD	float	1	28.665599999996516
v	float64	(20, 20)	[[-0.1881457 0.19628134 -0.15382849 ... -0.007987 -0.01547316 0 ...
w	float64	(20,)	[5.81855199e+08 1.47845483e+05 7.47652177e+03 ... 4.05506265e+02 4.14 ...
zeta	float64	(16564, 20)	[[-0.00101288 0.0013909 0.00149613 ... 0.02024843 -0.00212133 -0 ...

Method	Complexity (flops)
SVD	$\mathcal{O}(n^2m + nm^2 + m^3)$
MOS	$\mathcal{O}(nm^2 + rnm + m^3)$

Reference: *Wang, Zhu & Mcbee, Brian & Iliescu, Traian. (2015). Approximate Partitioned Method of Snapshots for POD. Journal of Computational and Applied Mathematics. 10.1016/j.cam.2015.11.023.*

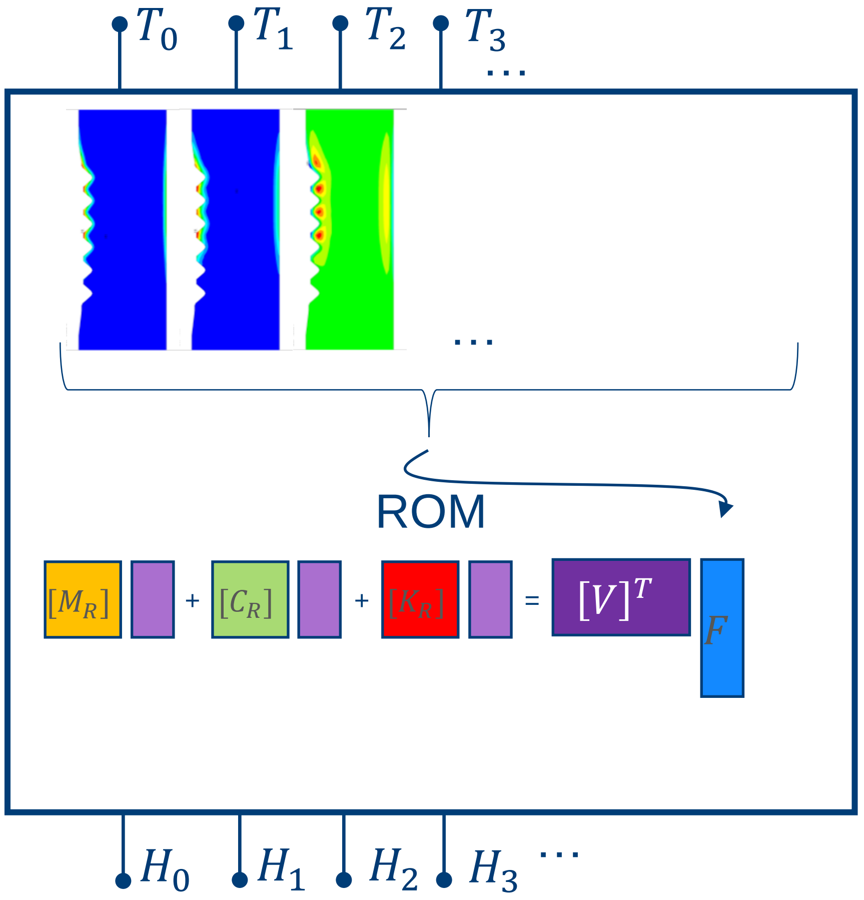
Source: Master thesis Hamza Jamil „ Model Order Reduction – Induction Hardening Process”

ROM Generation: Load vectors == modes

CADFEM

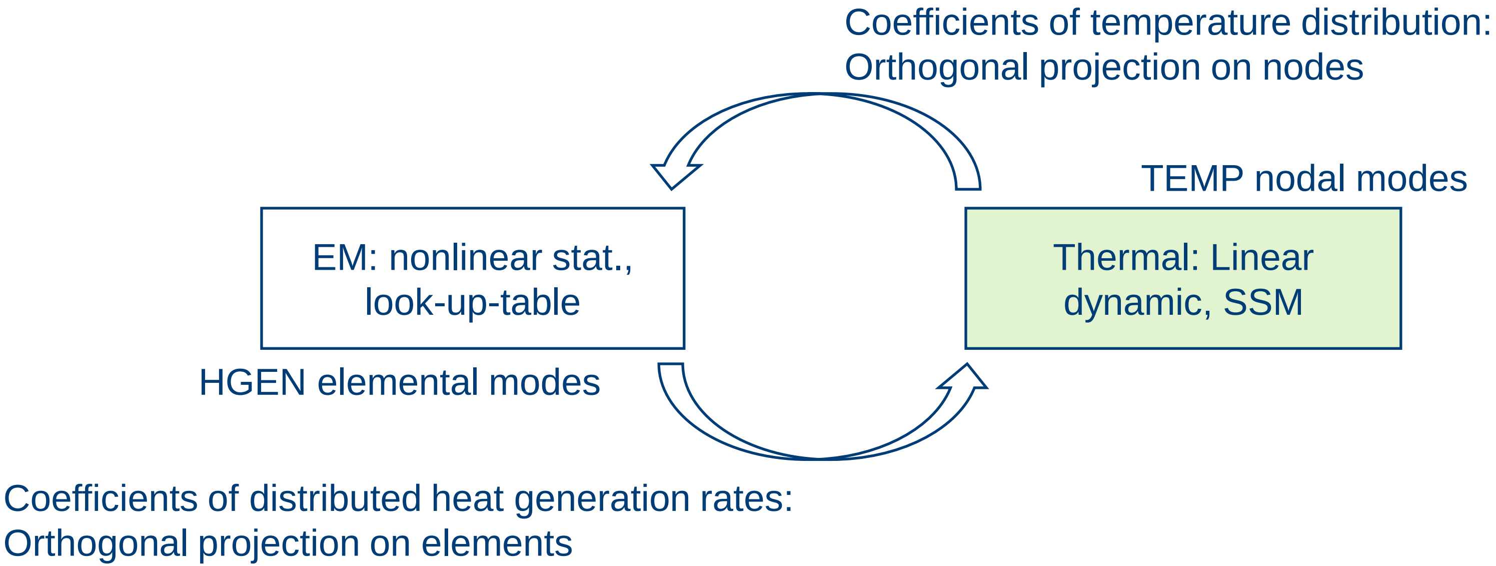
Two step reduction process:

- Reduction of field distribution
  - DOF characterized by a small set of functions / basis vectors
- Reduction of bulk matrices
- Projection onto Krylov Subspace
  - Load vectors: Linear combination of basis vectors
  - Description of dynamic relation between linear combination of heat generation basis vectors and temperature basis vectors



Which part of the solution is in place?

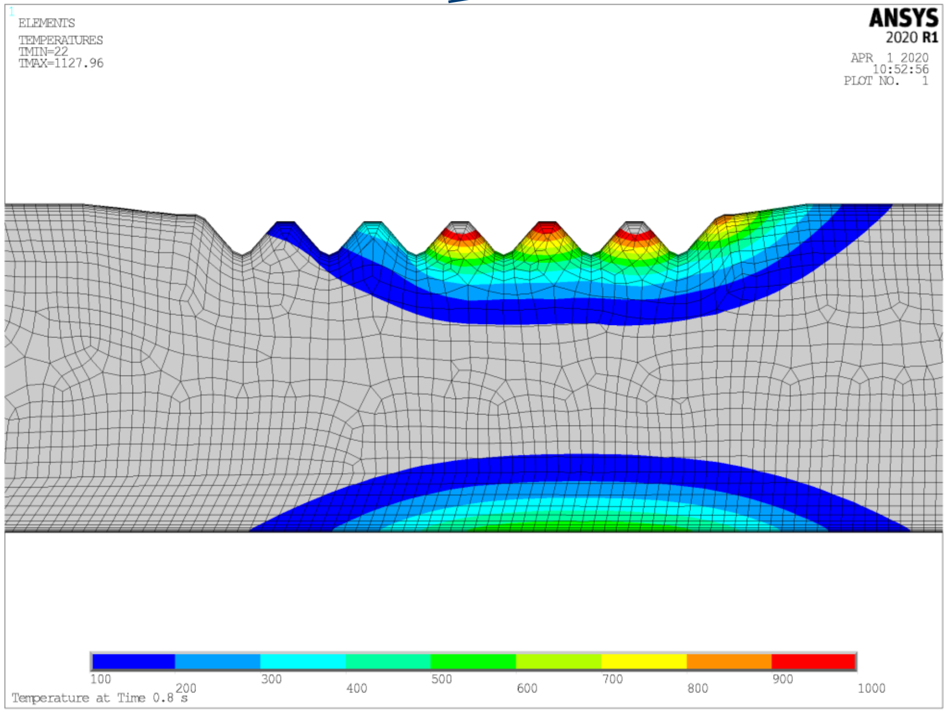
CADFEM



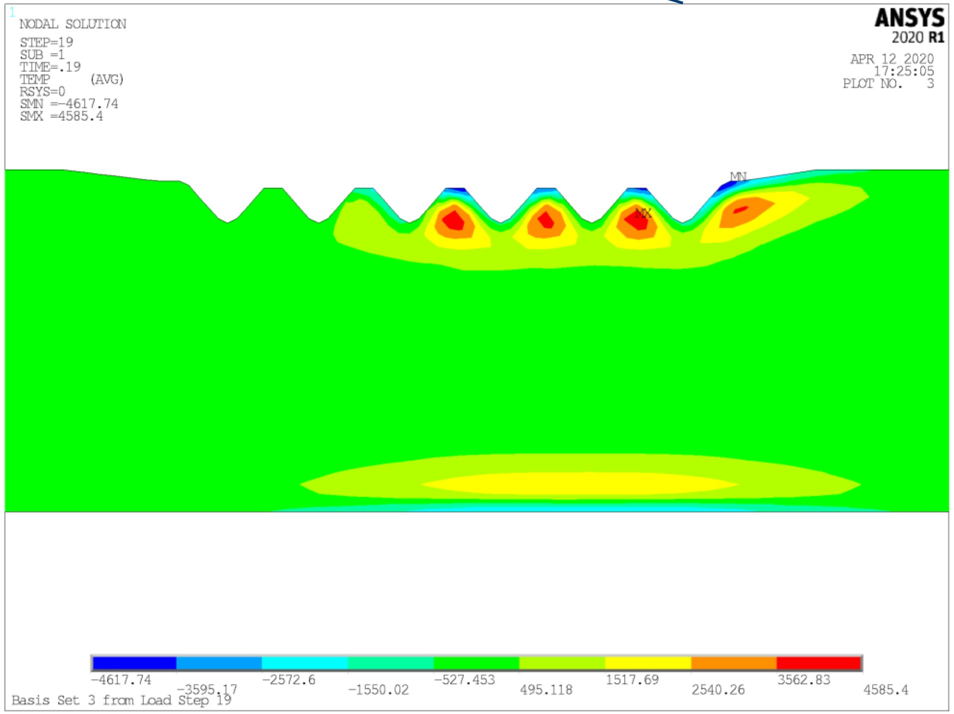
Determination of TEMP and HGEN coefficients

CADFEM®

$$c_{T3}(t = 1127s) = \langle TEMP(t = 1127s), mode\_3 \rangle$$



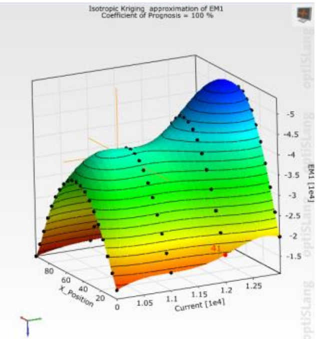
*



Response surface generation

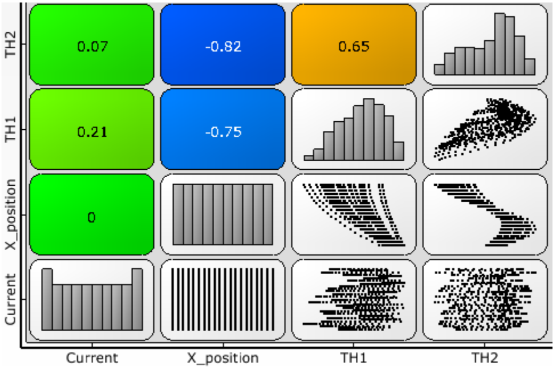
CADFEM®

- Inputs:
  - Current
  - Inductor position
  - TEMP coefficients
- Outputs:
  - HGEN coefficients



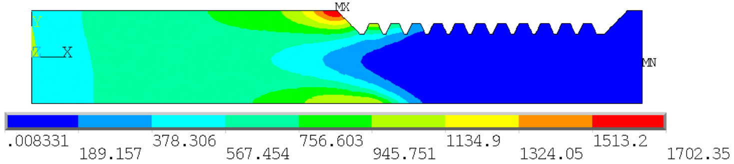
- Approximation Method: Kriging

- Quality of response surface is influenced by:
  - Number of Modes
  - Number of samples
  - Space filling of samples: Parameter spread
  - DOE: Based on Energy → input parameters derived from technologically achievable design spaces

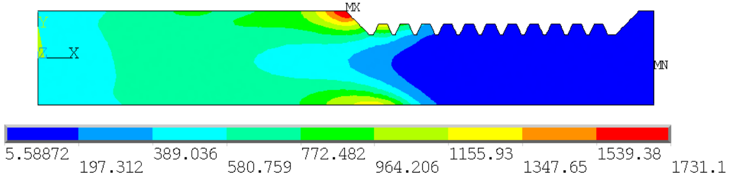


Temperature field for untrained current value of 12500A @ final time

FEM



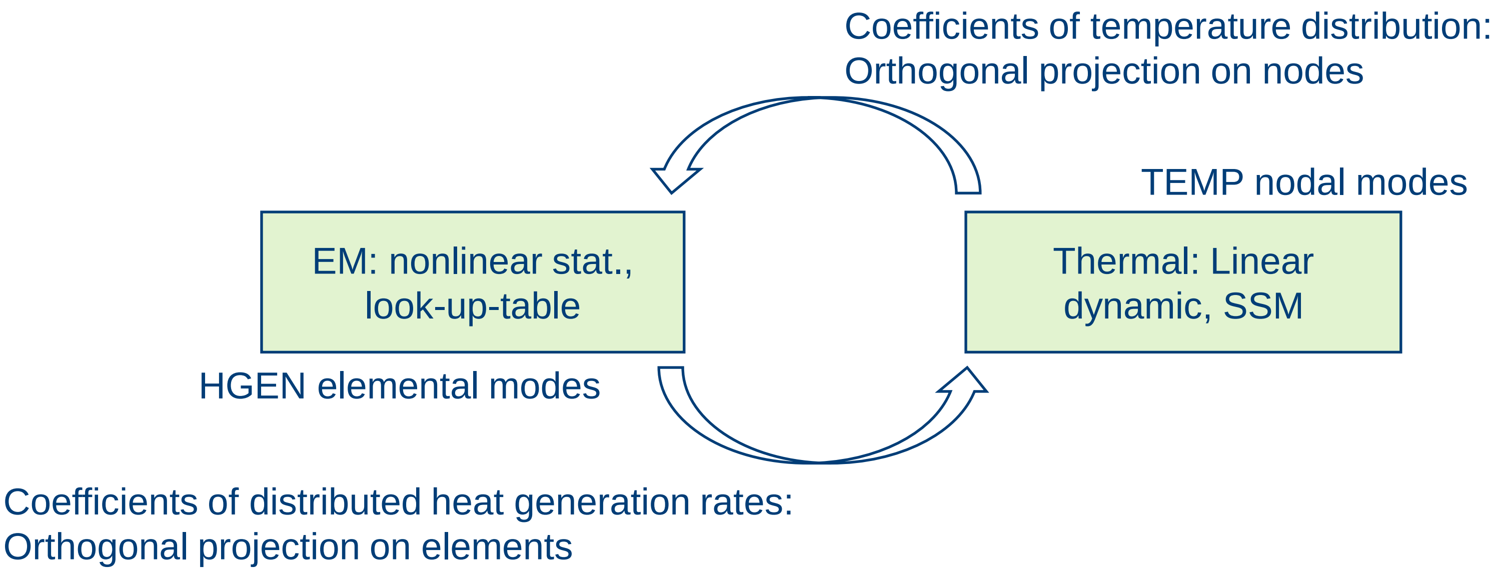
ROM



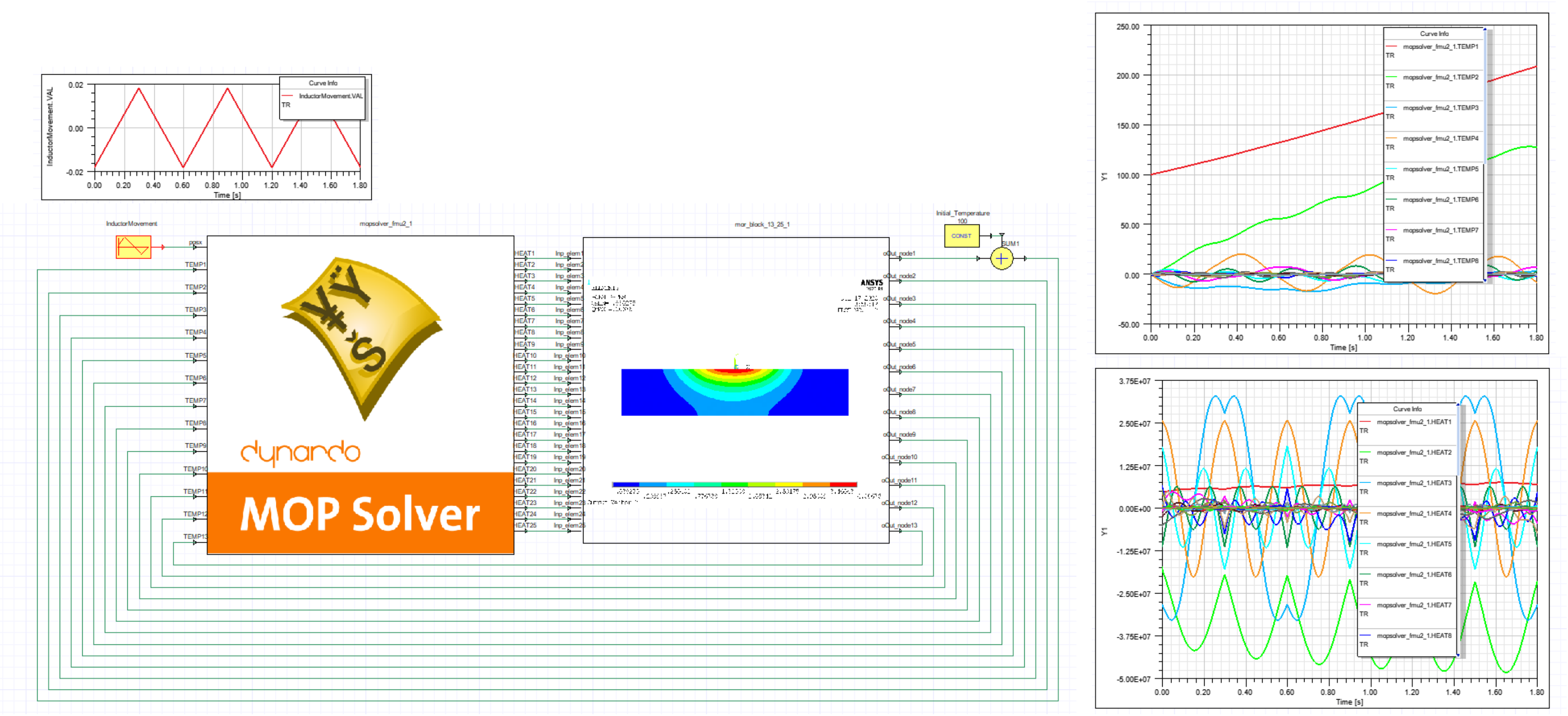
Error Norm: 6.86%

Source: Hamza Jamil „ Model Order Reduction – Induction Hardening Process", MORSS 8.9.2020

Which part of the solution is in place?

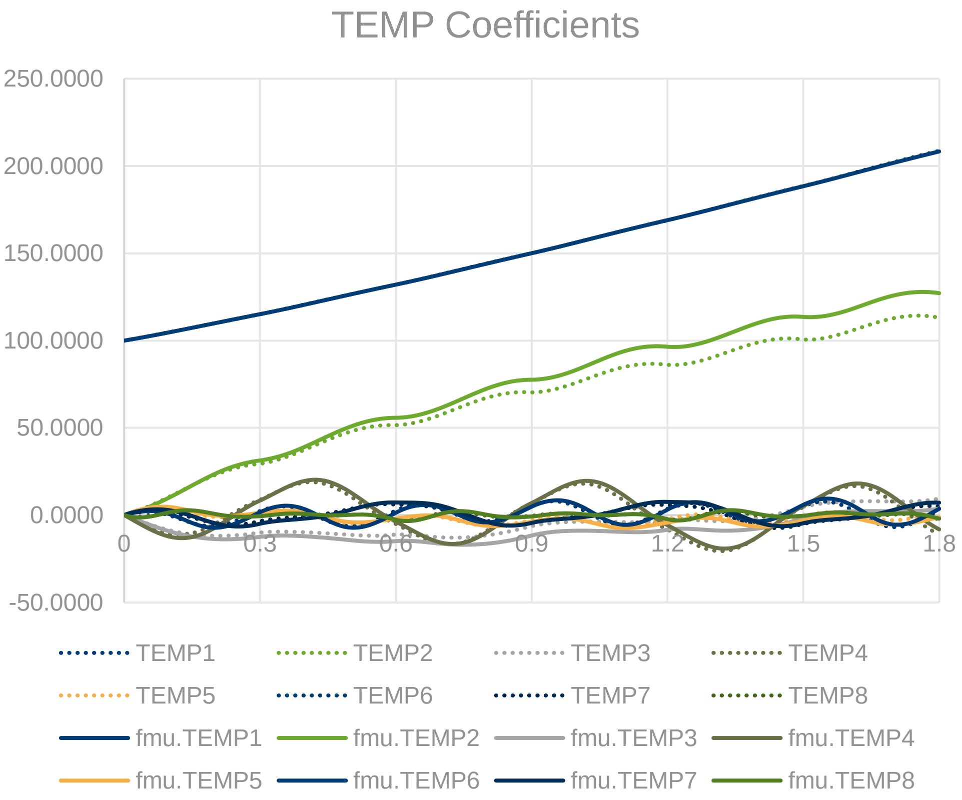


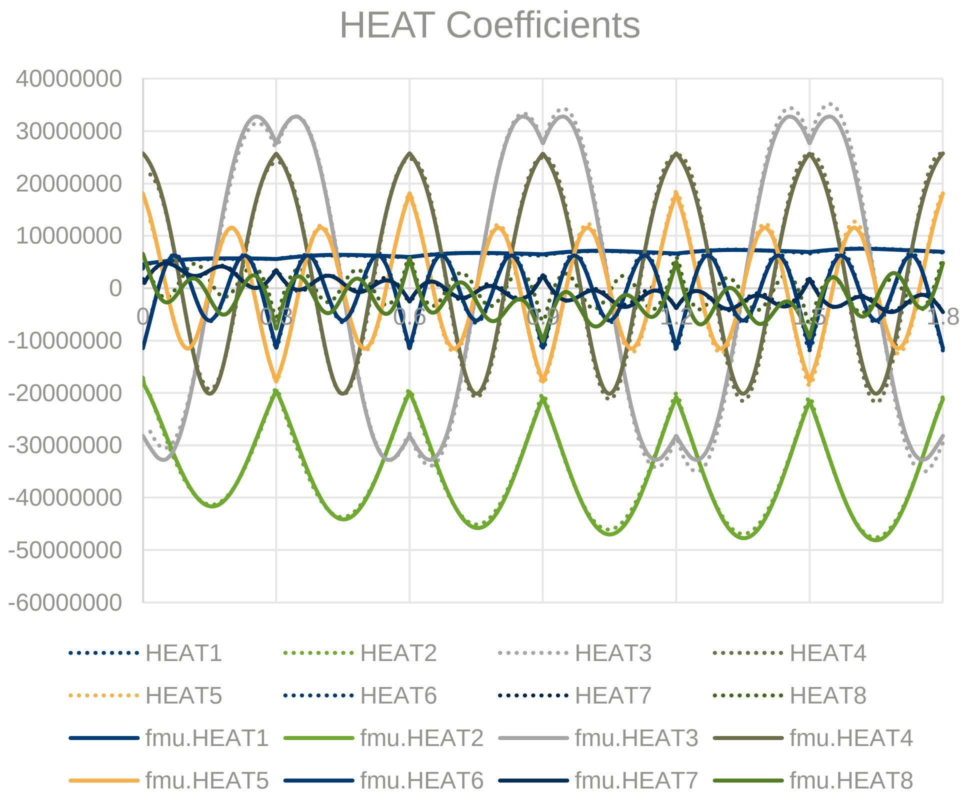
Setup on system level



Validation strategy – on system level

Comparison of coefficients derived from field and system simulation





Validation strategy – field error norm

CADFEM

- Validation simulation – data not applied for training
- Compare results from:
  - FEM-Solution
  - Expanded field data from system simulation with same reference load scenario

$$\Delta T(t = n, x, y, z) = \Delta T_n$$

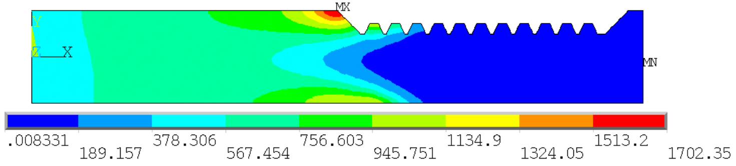
$$= T_{FEM}(t = n, x, y, z) - \sum_i c_i\,(t = n) \cdot T_i\,(x, y, z)$$

$$norm_{error} = \sqrt{\sum_n \Delta T_n^2 \cdot w_{node}(n)}$$

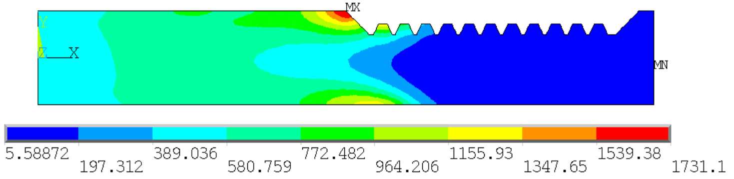
$$Percentage\ norm_{error} = \frac{norm_{error}}{norm_{FEM}} \cdot 100$$

Temperature field for untrained current value of 12500A @ final time

FEM



ROM



Error Norm: 6.86%

Summary and outlook for inductive hardening workflow:

- Goal: Error norm of reduced model below 10% achieved
- A progressive hardening process can be expressed effectively in a reduced system
- Speedup of 500 for 2D testcase (full transient FEM simulation vs. system simulation)

- Extension of method to 3D Models
- Method for appropriate definition of training data to be defined:
  - Minimize number of training runs required
  - Automatized generation of DOE

Conclusion

Reduced order models for nonlinear, transient problems with field interaction

- Issue:
  - Coupled models
  - Nonlinear and transient
  - Field quantities

- Solution:
  - Partitioning nonlinear and transient behavior: Response surface and state space model
  - Transition between field solution and terminals by projection/expansion with basis functions.

- Opportunities:
  - Solving a whole new class of system-level problems